

5-7

Study Guide and Intervention (continued)

Rational Exponents

Simplify Expressions All the properties of powers from Lesson 5-1 apply to rational exponents. When you simplify expressions with rational exponents, leave the exponent in rational form, and write the expression with all positive exponents. Any exponents in the denominator must be positive integers

When you simplify radical expressions, you may use rational exponents to simplify, but your answer should be in radical form. Use the smallest index possible.

Example 1

Simplify $y^{\frac{2}{3}} \cdot y^{\frac{3}{8}}$.

$$y^{\frac{2}{3}} \cdot y^{\frac{3}{8}} = y^{\frac{2}{3} + \frac{3}{8}} = y^{\frac{25}{24}}$$

Example 2

Simplify $\sqrt[4]{144x^6}$.

$$\begin{aligned}\sqrt[4]{144x^6} &= (144x^6)^{\frac{1}{4}} \\ &= (2^4 \cdot 3^2 \cdot x^6)^{\frac{1}{4}} \\ &= (2^4)^{\frac{1}{4}} \cdot (3^2)^{\frac{1}{4}} \cdot (x^6)^{\frac{1}{4}} \\ &= 2 \cdot 3^{\frac{1}{2}} \cdot x^{\frac{3}{2}} = 2x \cdot (3x)^{\frac{1}{2}} = 2x\sqrt{3x}\end{aligned}$$

Exercises

Simplify each expression.

1. $x^{\frac{4}{5}} \cdot x^{\frac{3}{5}}$

$$x^{\frac{4}{5}} \cdot x^{\frac{3}{5}} = x^2$$

2. $(y^{\frac{2}{3}})^{\frac{3}{4}}$

$$y^{\frac{2}{3} \cdot \frac{3}{4}} = y^{\frac{2}{4}} = y^{\frac{1}{2}} = \sqrt{y}$$

3. $p^{\frac{4}{5}} \cdot p^{\frac{7}{10}}$

$$p^{\frac{4}{5} + \frac{7}{10}} = p^{\frac{8}{10} + \frac{7}{10}} = p^{\frac{15}{10}} = p^{\frac{3}{2}} = p\sqrt{p}$$

$$m^{-\frac{12}{25}} \cdot \frac{1}{\sqrt[25]{m^{12}}} \cdot \sqrt[25]{m^{13}} = \frac{\sqrt[25]{m^{13}}}{m}$$

5. $x^{-\frac{8}{3}} \cdot x^{\frac{4}{3}}$

$$x^{-\frac{8}{3} + \frac{4}{3}} = x^{-\frac{4}{3}} = \frac{1}{\sqrt[3]{x^4}}$$

6. $(s^{-\frac{1}{6}})^{-\frac{4}{3}}$

$$s^{\frac{1}{6} \cdot \frac{4}{3}} = s^{\frac{2}{9}} = \sqrt[9]{s^2}$$

7. $\frac{p}{p^{\frac{1}{3}}}$

$$\frac{p}{p^{\frac{1}{3}}} = p^{\frac{2}{3}} = \sqrt[3]{p^2}$$

8. $(a^{\frac{2}{3}})^{\frac{2}{5}} \cdot (a^{\frac{2}{5}})^{\frac{3}{2}}$

$$a^{\frac{2}{3} \cdot \frac{2}{5}} \cdot a^{\frac{2}{5} \cdot \frac{3}{2}} = a^{\frac{4}{15}} \cdot a^{\frac{3}{5}} = a^{\frac{4}{15} + \frac{9}{15}} = a^{\frac{13}{15}}$$

9. $\frac{x^{-\frac{1}{2}}}{x^{-\frac{1}{3}}}$

$$\frac{x^{-\frac{1}{2}}}{x^{-\frac{1}{3}}} = x^{-\frac{1}{2} + \frac{1}{3}} = x^{-\frac{1}{6}} = \frac{1}{\sqrt[6]{x}}$$

10. $\sqrt[6]{128} \cdot 2^7$

$$2\sqrt[6]{2} \cdot 2^7 = 2^{\frac{1}{3}} \cdot 2^7 = 2^{\frac{22}{3}}$$

11. $\sqrt[4]{49}$

$$\sqrt[4]{49} = 7^{\frac{1}{2}} = \sqrt{7}$$

12. $\sqrt[5]{288}$

$$2\sqrt[5]{9}$$

13. $\sqrt{32} \cdot 3\sqrt{16}$

$$4\sqrt{2} \cdot 12 = 48\sqrt{2}$$

14. $\sqrt[3]{25} \cdot \sqrt{125}$

$$5^{\frac{2}{3}} \cdot 5^{\frac{3}{2}} = 5^{\frac{4}{3} + \frac{9}{6}} = 5^{\frac{17}{6}}$$

15. $\sqrt[6]{16}$

$$\sqrt[6]{2^4} = 2^{\frac{2}{3}} = \sqrt[3]{4}$$

16. $\frac{x - \sqrt[3]{3}}{\sqrt{12}}$

17. $\sqrt[3]{48}$

$$25\sqrt[3]{5}$$

18. $\frac{a\sqrt[3]{b^4}}{\sqrt{ab^3}}$

$$\sqrt[3]{4}$$

